

A Continuation to Goldbach's Strong Conjecture

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ABSTRACT: In this paper, we will provide an additional theorem, or a continuation to the Goldbach's Strong Conjecture (called the "strong" or "binary").

KEYWORDS: Prime number formula, Goldbach's Strong Conjecture

I. INTRODUCTION

The Goldbach's Strong Conjecture states that every even integer greater than 2 can be expressed as the sum of two primes. In this paper, we will provide a continuation to the Goldbach's Strong Conjecture. We will use a general formula for primes (>2)—prime number formula, and then find the general formula for the difference between them.

II. METHODS

Theorem/Statement: Every even integer can be expressed as the difference of two primes.

Proof:

We know that every prime number, $p (>2)$, can be expressed as:

$$p = (4k + 1) \text{ or } (4k - 1) = (4k \pm 1); \text{ where } k \in I^+[1]$$

Let there be 2 prime numbers (>2), namely, p_1 and p_2 .

$$\therefore p_1 = 4n \pm 1; n \in I^+$$

$$p_2 = 4m \pm 1; m \in I^+$$

$$\Rightarrow p_1 - p_2 = (4n \pm 1) - (4m \pm 1)$$

Thus, we have only 4 possibilities:

I. $(4n + 1) - (4m + 1)$

II. $(4n + 1) - (4m - 1)$

III. $(4n - 1) - (4m + 1)$

IV. $(4n - 1) - (4m - 1)$

Here, possibilities I and IV (same sign of 1s), and II and III (opposite sign of 1s) are similar.

CASE I: Considering possibilities I and IV

Therefore,

$$p_1 - p_2 = (4n + 1) - (4m + 1) \text{ or } p_1 - p_2 = (4n - 1) - (4m - 1)$$

$$\Rightarrow p_1 - p_2 = 4(n - m)$$

$$\Rightarrow p_1 - p_2 = 2[2(n - m)]$$

$$\Rightarrow p_1 - p_2 = 2f; \text{ where } f = [2(n - m)]$$

Thus, f is an even integer (\because it is of the form $2q$)

$$\therefore f \in \{ \dots, -2, 0, 2, \dots \}$$

CASE II: Considering possibilities II and III

Therefore,

$$p_1 - p_2 = (4n + 1) - (4m - 1) \text{ or } p_1 - p_2 = (4n - 1) - (4m + 1)$$

$$\Rightarrow p_1 - p_2 = 4(n - m) \pm 2$$

$$\Rightarrow p_1 - p_2 = 2[2(n - m) \pm 1]$$

$$\Rightarrow p_1 - p_2 = 2f'; \text{ where } f' = [2(n - m) \pm 1]$$

Thus, f' is an odd integer (\because it is of the form $2q \pm 1$)

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$\therefore f' \in \{ \dots, -1, 1, 3, \dots \}$

Hence, on combining cases I and II, we obtain

$$p_1 - p_2 = 2F ; \text{ where } F \in f \cup f', \text{ i.e. } F \in I \\ \therefore F \in \{ \dots, -2, -1, 0, 1, 2, \dots \}$$

This means that every even integer ($2F$

$\in \{ \dots, -4, -2, 0, 2, 4, \dots \}$) will have atleast one pair of primes (p_1 and p_2) such that $p_1 - p_2 = 2F$

Here, we have 3 cases:

I. $p_1 > p_2 \Rightarrow p_1 - p_2 > 0 \Rightarrow p_1 - p_2$ will contain all the positive even integers

II. $p_1 = p_2 \Rightarrow p_1 - p_2 = 0$

III. $p_1 < p_2 \Rightarrow p_1 - p_2 < 0 \Rightarrow p_1 - p_2$ will contain all the negative even integers

Therefore, the above 3 cases cover all the even integers.

III. CONCLUSION

Thus, every even integer can be expressed as the difference of two primes.

REFERENCES

[1] Rushal Sohal—Proof for Goldbach's Strong Conjecture (http://www.irphouse.com/ijmr17/ijmrv9n2_04.pdf)